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Key Points:

- The developed image filtering algorithm has an advantage in SNR over adaptive and median filters both for Gaussian and non-Gaussian noise
- The best results of nonlinear filter operation are observed when images are broken by pulse noise with Johnson distribution
- RGB frame nonlinear filtering has an advantage over static JPEG filtering up to 5 dB for WGN and more than 10 dB for non-Gaussian noise

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Restoration of Static JPEG Images and RGB Video Frames by Means of Nonlinear Filtering in Conditions of Gaussian and Non-Gaussian Noise

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Abstract The use of nonlinear Markov process filtering makes it possible to restore both video stream frames and static photos at the stage of preprocessing. The present paper reflects the results of research in comparison of these types image filtering quality by means of special algorithm when Gaussian or non-Gaussian noises acting. Examples of filter operation at different values of signal-to-noise ratio are presented. A comparative analysis has been performed, and the best filtered kind of noise has been defined. It has been shown the quality of developed algorithm is much better than quality of adaptive one for RGB signal filtering at the same a priori information about the signal. Also, an advantage over median filter takes a place when both fluctuation and pulse noise filtering.

1. Introduction

Today, the filtration of video stream frames and restoration of scanned images are an actual task. The first stage of single block image restoration is a preprocessing, which allows enhancing the image quality by means of image processing methods such as filtering and noise reduction.

Preprocessing implies that an image is rectified from scanning defects. It can be achieved by standard methods of image processing, for example, various filters. However, filtering quality significantly depends on kind of noise distribution, whereas the most of existing filters are tuned only on statistical noise with Gaussian distribution.

One of the most popular filters used for image restoration is a linear filter, particularly, linear FIR (finite impulse response) filter. The filters of this type are quite effective computationally and simple to implement. However, in application to digital images they have a number of disadvantages: objects contours is blurred and small image parts can be lost.

These features are extrinsic to nonlinear filters, the simplest of which is a median filter. However, they have comparably lower efficiency against the fluctuation noise (Arce, 2005). Median filters give much better effect when processing the image distortion generated by impulse noise (≪scratches≫, failed lines, ≪touches≫, etc.).

At the same time the use of adaptive FIR filters allows saving the object contours in the image with the fluctuation noise (Methods of computer image processing (Gashnikov et al., 2003).

Therefore, the development of filter that is effective both for fluctuation and impulse noise is an actual problem. And we consider nonlinear Markov algorithm as an appropriate solution for image processing, which can significantly improve the filtering quality even in nonstationary mode as compared with the adaptive algorithm for the same a priori uncertainty. Markov algorithm makes it possible to use the same system of stochastic differential equations to describe the wide range of signals and interferences (Astretsov & Sokolov, 2014).

Thus, we propose to develop an algorithm of quasi-optimal reception based on Markov theory of nonlinear filtering. This algorithm will allow filtering static JPEG images and RGB signals in conditions of white Gaussian and non-Gaussian (Johnson) noise without loss of quality.

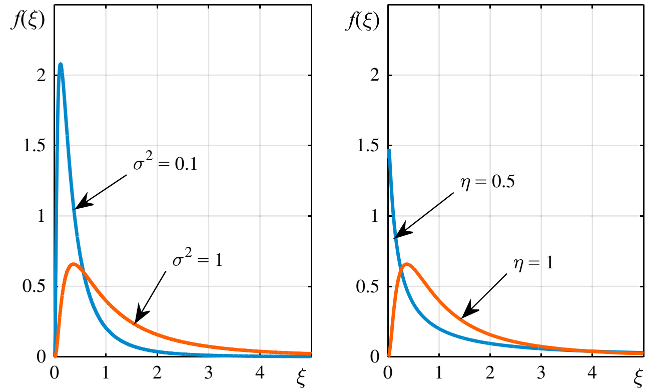


Figure 1. Probability density of S_L Johnson distribution at various values of variance σ^2 and η.

In the experimental part of research we compare the filtering quality of video stream RGB frame and frame converted to static JPEG digital image.

2. Synthesis of Quasi-Optimal Algorithm

The proposed algorithm is to perform the process of signal processing, when the signal is need to be received in the sum with noise. This algorithm is based on the following:

1. The signal is taken as a sequence of video pulses with 256 amplitude levels and unknown initial parameters (useful signal $\lambda(t)$, the amplitude value, duration, time of arrival, carrier frequency) and a priori defined as Markov process.
2. The noise $n(t)$ is a discrete random process, with one of three Johnson distributions (S_L, S_B, and S_U), which can be described as nonlinear transform of Markov Gaussian process $z(t)$: $n(t) = q\,(z(t))$.
   The noise of natural origin is believed to be a random process with Gaussian distribution (Astretsov & Sokolov, 2014). An internal receiver noise is described by normal distribution (random variable with zero mean and unitary standard deviation). At the same time we consider the noise of artificial origin as a random process with one of three Johnson distributions (S_L, S_B, or S_U) (Astretsov & Sokolov, 2014).
   Johnson has described the system of random variables, which are obtained by transforms of variables with normal distribution density. These transforms let approximate various distributions including distributions lumped on semiaxes or compacts. Therefore, it is possible to approximate the industrial noise distribution density with Johnson distributions, which depend on three or four parameters and, consequently, describe more broad class of densities, than two-parameters-dependable Gaussian distribution. Additionally, the random variables with Johnson distribution can be transformed to normal random variables. It provides the easiest way to carry out the statistic data processing. Figures 1–3 show the densities of random variables at different parameters values.
   The probability density of Gaussian distribution is defined as follows:

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(\xi - m_\xi)^2}{2\sigma^2}\right]. \tag{1}$$

   The probability density of S_L Johnson distribution is defined as follows:

$$f(\xi) = \frac{\eta}{\sqrt{2\pi}(\xi - m_\xi)} \exp\left[-\frac{1}{2}\eta^2 \left(\frac{\gamma}{\eta} + \ln\frac{(\xi - m_\xi)}{\sigma}\right)^2\right]. \tag{2}$$

   The random variables z with Gaussian distribution are expressed through the values of S_L Johnson distribution:

$$z = \gamma + \eta \ln\frac{\xi - m_\xi}{\sigma}. \tag{3}$$

   An expression for probability density of S_B Johnson distribution looks as follows:

$$f(\xi) = \frac{\eta}{\sqrt{2\pi}} \frac{\sigma}{(\xi - m_\xi)(\sigma - \xi + m_\xi)} \exp\left[-\frac{1}{2}\eta^2 \left[\frac{\gamma}{\eta} + \ln\left(\frac{\xi - m_\xi}{\sigma - \xi + m_\xi}\right)\right]^2\right]. \tag{4}$$

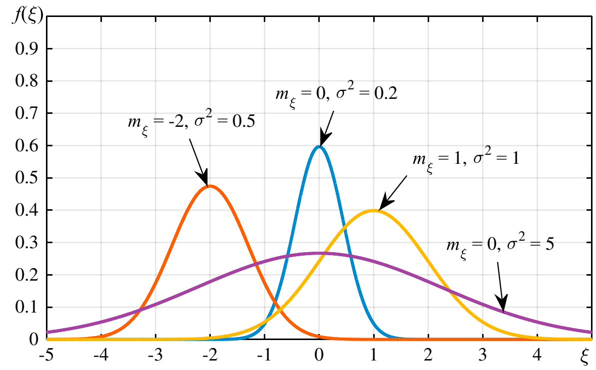


Figure 2. Probability density of Gaussian distribution at various values of variance σ^2 and mean m_ξ.

The random variable z with Gaussian distribution expressed through the values of S_B Johnson distribution:

$$z = \gamma + \eta \ln \frac{\xi - m_\xi}{\sigma - \xi + m_\xi}. \tag{5}$$

The probability density of S_U Johnson distribution is defined as follows:

$$f(\xi) = \frac{\eta}{\sqrt{2\pi\left(\sigma^2 + (\xi - m_\xi)^2\right)}} \exp\left[-\frac{1}{2}\eta^2\left[\frac{\gamma}{\eta} + \ln\left(\frac{\xi - m_\xi}{\sigma} + \sqrt{1 + \left(\frac{\xi - m_\xi}{\sigma}\right)^2}\right)\right]^2\right]. \tag{6}$$

The random variable z with Gaussian distribution expressed through the values of S_B Johnson distribution:

$$z = \gamma + \eta \ln\left(\frac{\xi - m_\xi}{\sigma} + \sqrt{1 + \left(\frac{\xi - m_\xi}{\sigma}\right)^2}\right). \tag{7}$$

3. The process performing the signal has a slow change in time compared with the change of the process describing the noise. (On two samples of signal must be at least 10 samples of interference.)
4. The signal and noise are nonstationary processes. So the input receiver signal is

$$y(t) = \lambda(t) + z(t), \tag{8}$$

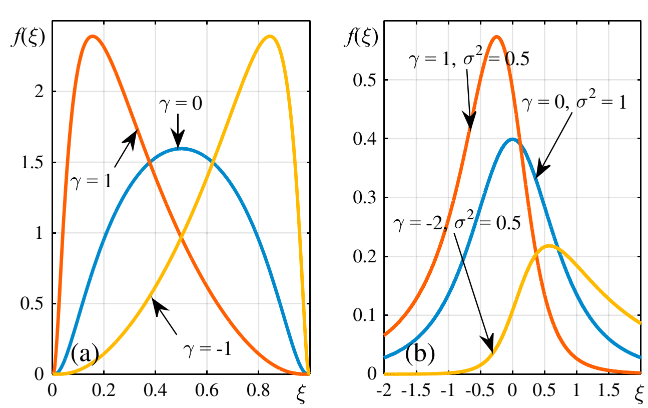


Figure 3. Probability density of (a) S_B and (b) S_U Johnson distributions.

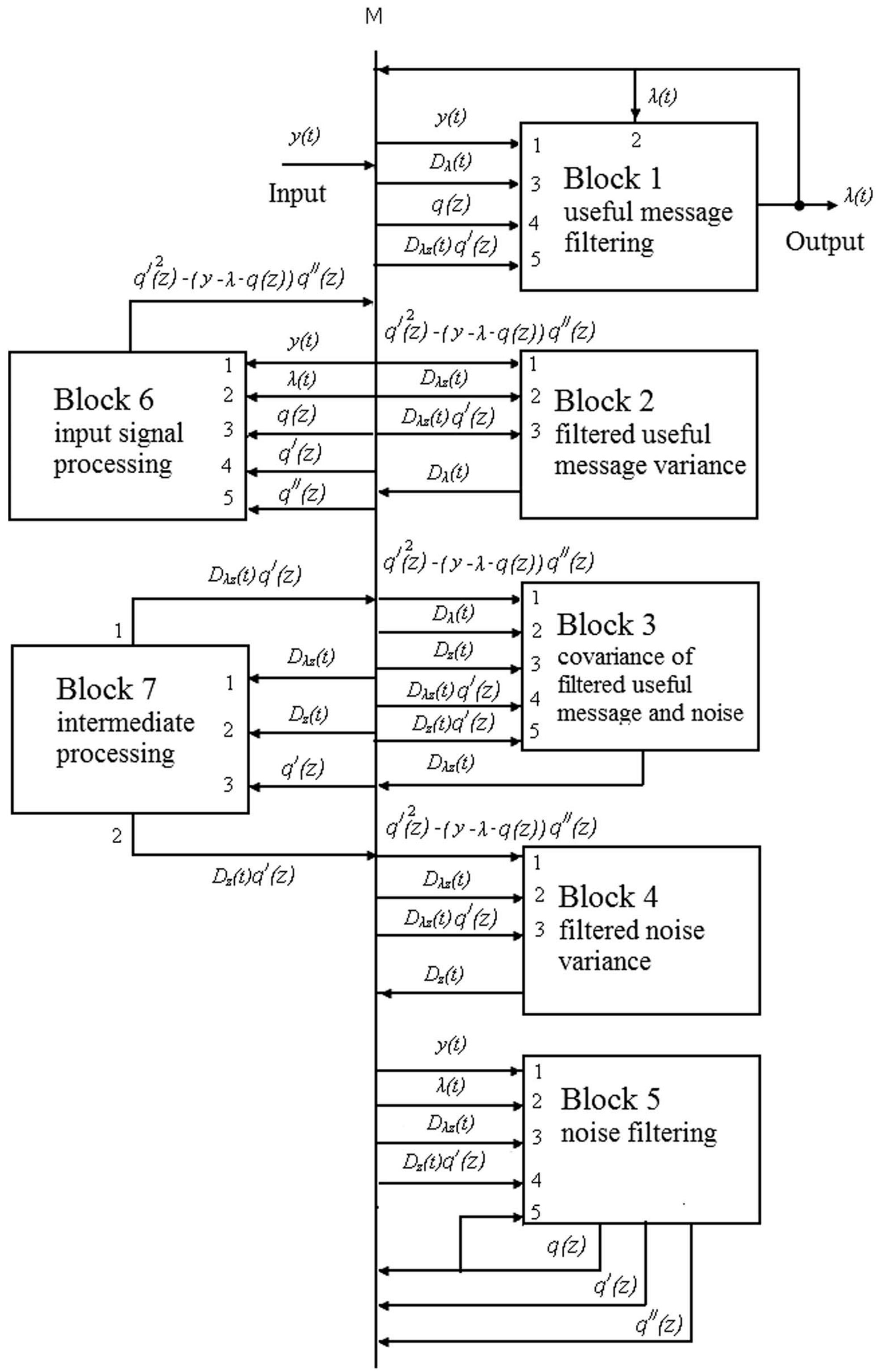


Figure 4. Generalized block diagram of quasi-optimal receiver based on nonlinear filtering.

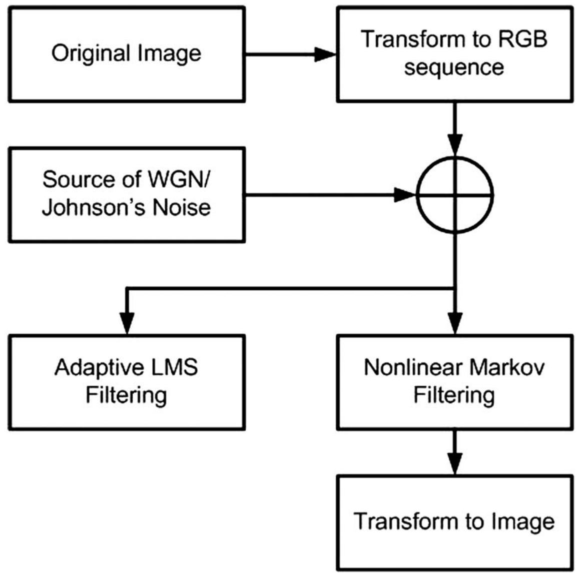


Figure 5. The block diagram of RGB video image processing in LabView.

where $\lambda(t)$ and $z(t)$ are the signal and noise both presented as Markov process (Pervachev, 1982). The signal $\lambda(t)$ is given by a priori stochastic differential equation:

$$\frac{\partial \lambda(t)}{\partial t} = -k_1 \lambda(t) + n_\lambda(t), \tag{9}$$

where k_1 is a linear coefficient proportional to the single video pulse duration; $n_\lambda(t)$ is a forming white noise with one-sided spectral density N_λ and an autocorrelation function

$$\langle n_\lambda(t_1) n_\lambda(t_2) \rangle = \frac{1}{2} N_\lambda \delta(t_2 - t_1). \tag{10}$$

At the same time the noise $z(t)$ is given by analogous a priori stochastic differential equation:

$$\frac{\partial z(t)}{\partial t} = -k_2 z(t) + n_z(t), \tag{11}$$

where k_2 is a linear coefficient inversely proportional to the noise sampling frequency; $n_z(t)$ is a forming white noise with one-sided spectral density N_z and an autocorrelation function

$$\langle n_z(t_1) n_z(t_2) \rangle = \frac{1}{2} N_z \delta(t_2 - t_1). \tag{12}$$

The video pulses repetition rate ω, and their phase φ are determined by appropriate stochastic differential equations. But since these parameters are just to specify the signal $\lambda(t)$, in order to obtain algorithm bounds, it is sufficient to solve integral-differential Stratonovitch equation for two-component two-dimensional Markov process (Tikhonov & Kulman, 1975):

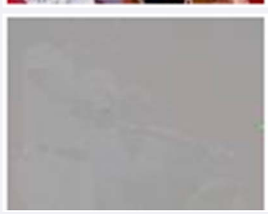
$$\frac{\partial W(t,\lambda)}{\partial t} = -\frac{\mathrm{d}}{\mathrm{d}\lambda}\left[a(\lambda,t)W(t,\lambda)\right] + \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\left[b(\lambda,t)W(t,\lambda)\right] + \left[F(t,\lambda) - \langle F(t,\lambda)\rangle\right] W(t,\lambda), \tag{13}$$

where $W(t,\lambda)$ is a posteriori probability density of the random process $\lambda(t)$; $a(\lambda,t)$ is a drift coefficient; $b(\lambda,t)$ is a diffusion coefficient; $F(t,\lambda)$ is an observation time derivative of the likelihood function logarithm. For an energy parameter $\lambda(t)$

$$F(t,\lambda) = \frac{1}{N_0}\left[y(t) - \lambda(t)\right]^2, \tag{14}$$

where N_0 is one-sided spectral density of noise.

Table 1
Filter Operation at Various Kinds of Noise When Processing RGB Video Image

			Filtered image	
Interference type	SNR (dB)	Image with noise	Adaptive filter	Nonlinear Markov filter
WGN	−10			
	5			
S_L Johnson interference	−5			
	−10			
S_B Johnson interference	−15			
S_U Johnson interference	−5			

A particular solution of (13) according to Tikhonov and Kulman (1975) is an equation system for quasi-optimal nonlinear filtering in transient mode:

$$\frac{\partial \lambda}{\partial t} = -k_1 \lambda + \frac{2}{N_0}(y - \lambda - q(z)) \left(D_\lambda + D_{\lambda z} q'(z)\right), \tag{15}$$

$$\frac{\partial z}{\partial t} = -k_2 z + \frac{2}{N_0}(y - \lambda - q(z)) \left(D_{\lambda z} + D_z q'(z)\right), \tag{16}$$

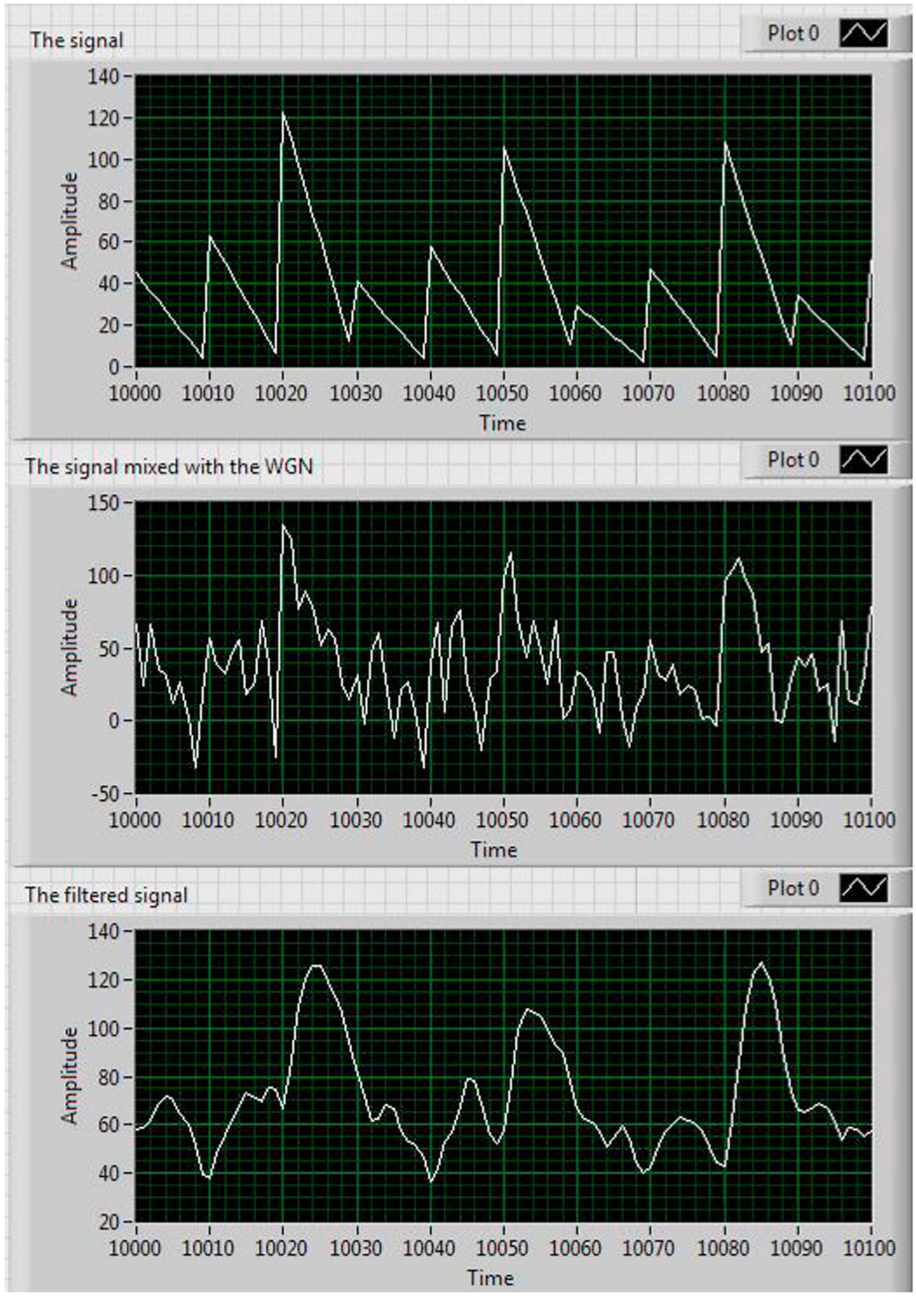


Figure 6. Oscilloscope traces emitted by RGB signal, signal and WGN mixture, and filter output signal.

$$\frac{\partial D_\lambda}{\partial t} = \frac{1}{2}N_\lambda - 2k_1 D_\lambda - \frac{2}{N_0}\left[D_\lambda^2 + 2D_\lambda D_{\lambda z} q'(z) + D_{\lambda z}^2\left(q'^2(z) - (y - \lambda - q(z))q''(z)\right)\right], \tag{17}$$

$$\frac{\partial D_z}{\partial t} = \frac{1}{2}N_z - 2k_2 D_z - \frac{2}{N_0}\left[D_z^2(q'^2(z) - (y - \lambda - q(z))q''(z)) + 2D_z D_{\lambda z} q'(z) + D_{\lambda z}^2\right], \tag{18}$$

$$\frac{\partial D_{\lambda z}}{\partial t} = \frac{1}{2}N_{\lambda z} - D_{\lambda z}(k_1 + k_2) - \frac{2}{N_0}\left[D_\lambda(D_{\lambda z} + D_z q'(z)) + D_{\lambda z} D_z\left(q'^2(z)(y - \lambda - q(z))q''(z)\right) + D_{\lambda z} q'(z)\right], \tag{19}$$

where λ and z are random Markov processes: λ is an estimated signal value, z is an estimated noise value; D_λ and D_z are variances of the signal $\lambda(t)$ and noise $z(t)$, respectively; $D_{\lambda z}$ is a covariance between the signal $\lambda(t)$ and noise $z(t)$ with one-sided spectral density $N_{\lambda z}$.

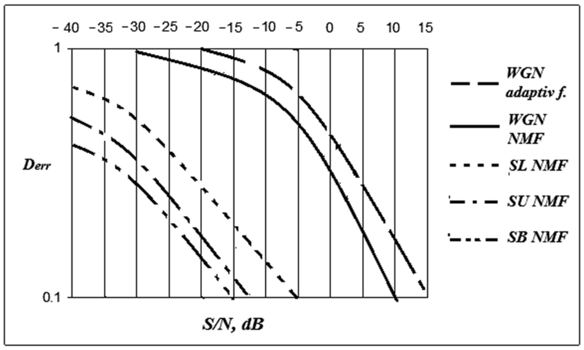


Figure 7. Error variance of correct detection versus SNR.

The system of equations (15)–(19) makes it possible to synthesize the algorithm for filtering of any video image under a priori uncertainty about the signal and noise. The system of five differential equations defines estimation of the signal $\lambda(t)$ and noise $z(t)$ values and determines the variance parameters D_λ and D_z, and its covariance $D_{\lambda z}$.

If white Gaussian noise (WGN) is considered as the interference instead of Johnson noise, the system of equations (8)–(12) is significantly simplified: $q(z)$ is replaced with z, and $q'(z)$ and $q''(z)$ are replaced with 1 and 0, respectively.

3. Filter Description

The technical result is achieved due to the construction of quasi-optimal receiver electrical circuit based on nonlinear two-component Markov process filtering in transient conditions. This circuit is obtained from conventional scheme by introduction of additional message processing blocks, which are responsible for filtering and extraction of additional signal $y(t)$ parameters: the useful message $\lambda(t)$, noise $z(t)$, variance $D_\lambda(t)$ of filtered useful message $\lambda(t)$, covariance $D_{\lambda z}(t)$ of filtered useful message $\lambda(t)$ and noise $z(t)$, and variance $D_z(t)$ of filtered noise $z(t)$. Additional blocks make it possible to extract the signal and noise parameters and issue them to the useful message filtering block in feedback circuits.

To solve the task, we have developed quasi-optimal receiver, which contains the filtering block and useful signal quasi-coherent reception block. The last one differs from conventional prototype (Astretsov & Sokolov, 2017) in that it contains seven blocks (Figure 4): (1) useful message $\lambda(t)$ filtering block, (2) extraction of filtered useful message variance $D_\lambda(t)$, (3) extraction of covariance $D_{\lambda z}(t)$ of filtered useful message $\lambda(t)$ and noise $z(t)$, (4) extraction of filtered noise variance $D_z(t)$, (5) noise $z(t)$ filtering block, (6) input signal $y(t)$ processing block, and (7) intermediate processing block of message and noise parameters.

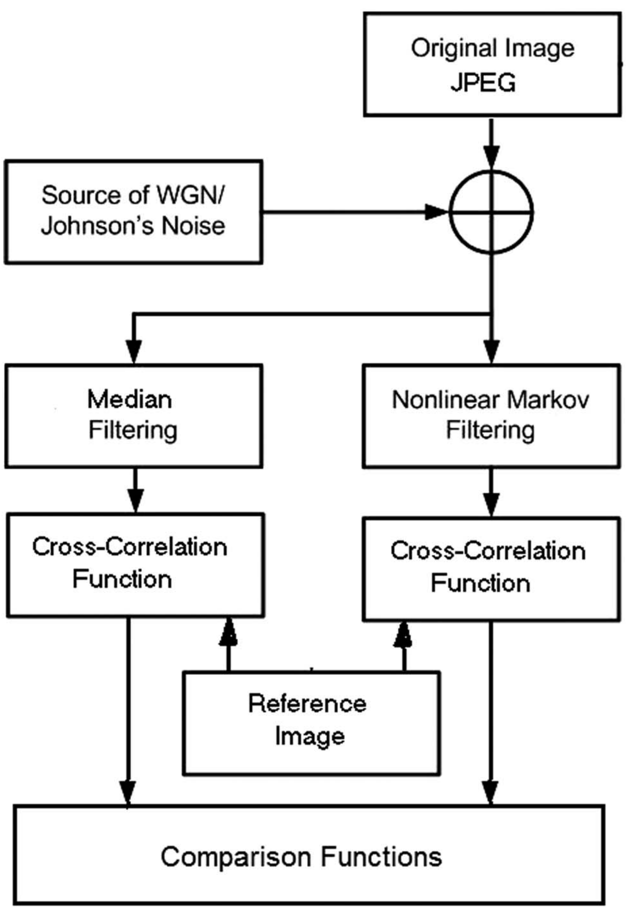


Figure 8. The block diagram of image processing in LabView.

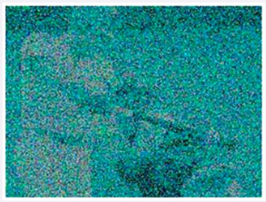
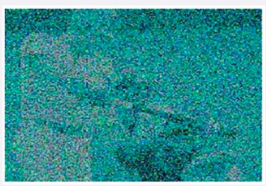
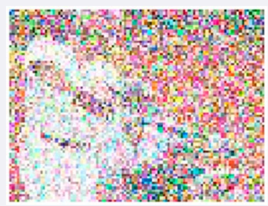
The channels are connected by the following links. Input mixture $y(t)$ of useful message and noise goes to the first inputs of blocks 1, 5, and 6. The output of block 1 is the output of quasi-optimal filter, which has a feedback with second inputs of blocks 1, 5, and 6. The output of fifth block nonlinear transformer $q(z)$ is connected with the inputs 4, 3, and 5 of blocks 1, 6, and 5, respectively. The output of fifth block nonlinear transformer $q'(z)$ is connected with the inputs 4 and 3 of blocks 6 and 7, respectively. The output of fifth block nonlinear transformer $q''(z)$ is connected only with the input 5 of block 6. The output of input signal processing block 6 is connected with the first ports of blocks 2–4. From the output of block 2 signal goes to the ports 3 and 2 of blocks 1, and 3, respectively. The output of the third block is connected with the first and third ports of blocks 7 and 5, as well as the second inputs of blocks 2 and 4. The fourth block output is linked with the third and second inputs of blocks 3 and 7. The intermediate processing block 7 output is linked with ports 5 and 4 of blocks 1 and 3, respectively, as well as the third ports of second and fourth blocks. The second output of intermediate processing block 7 is connected with inputs 5 and 4 of blocks 3 and 5.

4. RGB Video Signal Filtering

A digital experiment has been performed in order to determine the quality of nonlinear Markov filtering (NMF) algorithm in comparison with the adaptive one.

The digital model of NMF based quasi-optimal filter for RGB signal processing has been implemented in LabView software (Figure 5). We used the standard block from the LabView library as the adaptive filter to compare

Table 2
Filter Operation at Various Kinds of Noise When Processing Static Image

			Filtered image	
Interference type	SNR (dB)	Image with noise	Median filter	Nonlinear Markov filter
Original image				
WGN	0			
S_L Johnson interference	−5			
S_B Johnson interference	−15			
S_U Johnson interference	−5			

the results. This block creates an adaptive FIR filter with the standard least mean squares algorithm. The filter length has been taken as 32. The step size has been chosen as 0.002.

During the experiment we used a 256-level pulse sequence as a model of RGB signal. Two *.jpg* format images with dimensions of 200 × 160 pixels and 100 × 75 pixels has been chosen for processing. For both experiments an information samples repetition rate has been taken 10 times less than the noise sampling frequency.

The results of proposed algorithm operation in comparison with results of adaptive filtering are presented in Table 1. It shows the image mixed with WGN or the other noises with Johnson distributions and visual results of their processing by means of adaptive and nonlinear Markov filters at different values of signal to noise ratio (SNR). Obviously, the quality of nonlinear Markov filtering is much better when non-Gaussian noises acting, because the standard adaptive filter is not for the interferences with the translated momentary value distribution. However, developed NMF also has a feature: for the small values of SNR the recovery of clear boundaries in the image leads to the color distortion, particularly, considered images have been inverted to the green range.

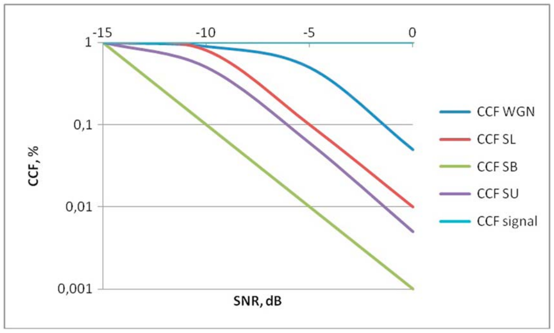


Figure 9. Dependencies of NCCF versus SNR for various types of noise.

The reason that the image colors invert is the filter inertness. On the one hand, the RGB signal bits sequence is well known to be defined by strict sequence of colors: each pixel is described by three bits (R: red, G: green, and B: blue). On the other hand, we improve the filtering quality adjusting the value of coefficient k_1 in range from 0.1 to 0.5. That is why we observe the strong smoothing of initial 3 bit portions per pixel in the signal and noise mixture. And since the first bit of each portion defines the red color intensity, then it is the subject of the great distortion with a shift in time. The resulting bit sequence of filtered signal is presented in third oscilloscope trace of Figure 6. With reference to Figure 6 it is impressive to see that the peaks in filtered signal are shifted in time by four samples against the peaks of original pulses presented in the first waveform. Therefore, the maximum pulse amplitude does not come to the first ≪red≫ bit, but only to the second ≪green≫ bit.

We used the square of filtering error D_{err} (an error variance) as a criterion of the filtering quality. As a study result, we have obtained the plots demonstrating the error variance of correct detection D_{err} versus SNR for different types of interference and parameters of their distributions (Figure 7).

5. Static JPEG Image Filtering

The digital model of NMF-based quasi-optimal filter for static JPEG image processing has also been implemented in LabView software (Figure 8). We used the standard median filter (right rank is −1 and left rank is 5) from the LabView library for comparison of filtering quality.

The results of image processing by means of developed nonlinear Markov filter in comparison with median one are presented in Table 2. Like Table 1 it contains the image mixed with WGN or Johnson noise at different values of SNR and visual results of processing.

During the digital experiment we estimated the maximum value of cross-correlation function (CCF) between noisy image signal and reference noise-free image in dependence to SNR at the receiver input. The noisy image signal is defined by three-dimensional matrix of intensity level values for every single point of spatial grid. As the noise we used white Gaussian noise and Johnson noises with various parameters γ and η. The normalized value of CCF between reference signal and noise has been taken as a criterion of the filtering quality. As 100% we took the CCF between reference signal and signal mixed with noise.

As a study result, we have obtained dependencies of the CCF versus SNR for different types of interference with various parameters of their distributions. These results are presented in Figure 9.

6. Conclusion

The image processing algorithm has been synthesized on the basis of nonlinear Markov filter. It has been shown the developed algorithm works successfully both in conditions of white Gaussian and artificial non-Gaussian noises, which is confirmed by the experiments with the digital model. Additionally, experiments allow making the following conclusions:

1. The gain in SNR of nonlinear Markov filtering algorithm compared with one of the adaptive algorithms is from 5 dB for WGN and error variance D_{err} of correct detection equal to 0.1 when RGB signal processing.
2. The best results of RGB frame processing are observed against the pulse noise with S_L Johnson distribution. The worst results are under the fluctuation noise.
3. For the small values of SNR the restoration of RGB frame leads to the color distortion. Particularly, considered images has been inverted to the green range because of the filter inertness. Significant improvement of the filtering quality and image boundaries identification is achieved by means of using the filtering coefficient k_1 values in range from 0.1 to 0.5.
4. The inversion of RGB frame to the green range is observed when the SNR value is equal to 0 dB under Gaussian and S_L Johnson noises. In case of S_U Johnson noise this situation takes a place when SNR = −5 dB. For S_U Johnson noise inversion does not occur up to the minimum tested values of SNR = −15 dB.
5. The gain in SNR of nonlinear Markov filtering algorithm compared with one of the median algorithms is from 5 dB for WGN and about 1−2 dB for Johnson noise when JPEG image processing.

6. The best results of static JPEG image processing are observed against the pulse noise with S_B Johnson distribution. The worst results are under the fluctuation noise.
7. Quasi-optimal receiver based on video stream frame nonlinear filtering has an advantage over static JPEG filtering up to 5 dB for WGN and more than 10 dB for non-Gaussian noise. The gain is due to the signal streams parallel processing in video frames, whereas static images have only one stream.

Acknowledgments
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